Are choices between risky options predictable?

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Abstract

The random utility model (RUM) is a fundamental notion in studies of human choices between risky options, pursued primarily within behavioural economics. The explanatory power of RUM however is undermined by the case-dependence of the choice function's free parameter. We address this limitation by contextualising utilities based on the concept of divisive normalisation, well-established in neural computation studies of decision-making. We derive a new model, the contextualised RUM (cRUM), with a new choice parameter β that linearly scales the normalised, rather than the raw, utility. The consequence, setting cRUM apart from RUM, is the independence of β on case-specific prospects, thereby facilitating predictions across experimental settings. We demonstrate that the cRUM prediction of variable framing effect among decision-makers in neuroeconomics studies aligns with the observed experimental data (with no meaningful difference between the medians). Moreover, 12 prospect choice experiments are predicted with cRUM yielding good agreement with true target labels particularly for gain/loss prospects (Pearson's correlation in the range of 0.70–0.95). Our results strongly suggest that cRUM strengthens the predictive capabilities of RUM, while providing a novel characterisation of the choice function in the neuro-cognitive context.

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Introduction

Predicting human choices is a challenging problem with implications well beyond behavioural economics [1, 2, 3, 4, 5]. Making a choice between alternatives with valued, still uncertain, outcomes (prospects) is expected to maximise the utility, typically expressed in monetary units, for the decision-maker. Predicting economic decisions in general and prospect choices in particular must somehow combine *exogenous* variables that are observable and *endogenous* (internal or latent) variables that are not directly observable and vary among decision-makers. In the classical formulation of random utility theory [6, 7], the utility of a prospect choice depends on both exogenous and endogenous variables, the latter typically represented as random variables and associated with bias or bounded rationality (subjectivity) [8]. Combining random utility theory with prospect theory (PT) [9] to define utilities (subjective valuations) yields a common form of the *random utility model* (RUM), which lends itself to predicting choice probability in an experimental setting [10]. Even though PT has introduced several robust variables for quantifying biases in subjective valuations, the predictive power of RUM is limited by the need to calibrate a free parameter of the choice function weighing the utilities [11, 12, 13, 14, 10], denoted here as β, on a set of observed choices for context-specific outcomes. The model can then only be applied in settings with comparable outcomes [10], i.e., it does not generalise beyond the given context. The variety of nomenclature used in the literature for the free calibration parameter in RUM (e.g., "rationality parameter", see Supplementary Information for diverse nomenclature used in the literature) indicates that β is understood as an endogenous parameter, yet in the canonical RUM formulation β is explicitly dependent on the magnitude of the utilities, i.e., on exogenous variables. The ambiguity of β dependence on exogenous *and* endogenous variables in RUM limits its applicability: predictions can only be made in settings where prospect choices have already somehow been explored facilitating case-specific β calibration. We argue that separation of exogenous and endogenous variables is imperative, not only for improving predictability of economic choices [8], but also for their interpretability.

To address this challenge we propose a novel modelling framework built on contextualisation of the RUM choice function, hence called *contextualised RUM* (cRUM). We define cRUM using a variant of the so-called divisive normalisation [15], which yields a new form of the parameter β of the choice function, referred to here as the choice (or control) parameter. Crucially, this new choice parameter should facilitate better predictions of prospect choices: in the cRUM framework β does not explicitly depend on the prospect utilities, as in the classical RUM formulation, and thus is considered *endogenous*.

Our main goal is to validate the predictability of cRUM in two consecutive steps, which consider independent data on discrete choice experiments and different types of experimental settings. In the first step, we consider discrete choice experiments exploring variability of choice probability among decision-makers in a population. In the cRUM framework this means that β as an endogenous parameter should vary among decision-makers in a population. In the second step, we consider discrete choice experiments across a population where choice probabilities are reported in the form of aggregated information across decision-makers. In the cRUM framework this means that even though β is expected to vary among decision-makers its mean, here interpreted as population-representative value, mean should model aggregated choices in a population of decision-makers.

This manuscript first introduces the novel model cRUM from RUM. The cRUM formulation strengthens the theoretical link between behavioural economics and psychology as well as sociology through the TPB framework, opening for validation and characterisation of the choice parameter β, in ways not attempted before. The proposed line of reasoning, in combination with human-social probability perception evidence, in particular perceptual numerosity experimental data and data on social perception of risk/unlikely events, is used in the paper to infer a stochastic model for β. We claim that a stochastic model for β in cRUM can capture variability of observed choice probability among a population of decision-makers. Moreover, we posit that cRUM can also predict observed choice probability across decision-makers and experimental settings in the simplified case where the mean of the β distribution is considered as a population-representative value. The cRUM with the proposed model for β is then tested, first showing that the stochastic model for β reproduces well the observed variability in experimental studies on variability of the framing effect in a population, and then illustrating a reasonably robust generalisation of β across experiments on the broadest set of prospect choice experimental data ever considered in a single study (to our best knowledge).

Results

From RUM to cRUM, a socio-psychological and neurobiological interpretation. The contextualisation proposed in cRUM suggests an overall structure consistent with the theory of planned behaviour (TPB) developed in the socio-psychological context [16]. Moreover, the proposed cRUM with divisive normalisation can be related to neural computations and thus interpreted in the neurocognitive domain [17].

The central notion of TPB is behavioural intention, which precedes overt behaviour or action; the expression *overt behaviour* $\sim$ *behavioural intention* implies a probabilistic relationship between valuations of options and choice in line with the S-shaped choice function in behavioural economics [10]. TPB asserts that behavioural intention, and consequently behaviour, is driven by three factors: attitude (utility resulting in a preference), perceived control (subjective evaluation of one's ability to achieve assessed utility by choosing a behaviour), and social factors. Human behaviour and prospect choices ultimately depend on valuation, i.e., appraisal of outcomes [6, 18, 8], to form an attitude toward alternative options [16]. Using the TPB framework, we can provide a comprehensive illustration of the concept of cRUM, building upon the foundation of RUM. Let P_A denote the probability of choosing prospect A over prospect B, and let V_A and V_B denote the raw utilities associated with A and B, respectively (Methods and Supplementary Information for details). In RUM, the "strong utility" or "Fechner choice function" defines $P_\text{A} = F(V_\text{A} - V_\text{B})$, where $F(\cdot)$ is an S-type function, e.g., Sigmoid or Logit, yielding $P_\text{A} = \big(1 + \exp(-\beta(V_\text{A} - V_\text{B}))\big)^{-1}$, and β is the free calibration parameter [11, 12, 13, 14, 10]. The novel model cRUM proposes a new method for computing the choice probability P_A and a context-independent choice parameter β, which does not depend on the utilities V_A and V_B. Fig. 1 (and Supplementary Fig. 1) illustrates how the cRUM can be set in the TPB framework. It starts from input information through valuation of outcomes to intention and, finally, probability of choosing one option over the other. The normalisation of utilities to define attitudes toward options A and B, i.e, $\zeta_\text{A} = V_\text{A}/V_n$ and $\zeta_\text{B} = V_\text{B}/V_n$ (with $V_n = |V_\text{A}| + |V_\text{B}|$), plays a central role in cRUM (Fig. 1): cRUM can be seen as an extension of the RUM with Fechner choice function that accounts for the contextualisation of valuation. The normalised utilities are then used to define the intention of a decision-maker to choose A over B, i.e., $\beta(\zeta_\text{A} - \zeta_\text{B})$, which operationalises the TPB framework of cRUM (eq. (1) and Methods for a detailed explanation).

The normalisation of outcomes is consistent with the finite representational bandwidth of the human brain that adapts the neural code to the range of outcome values in a given context; thereby adjusting the dynamic range so that all potential outcome values are mapped to the same range of the representative neural activity [15, 17, 19]. Although different normalisations such as average outcome variance [14, 20], absolute distance between value distributions [21], or difference between maximum and minimum outcome values [19] have been considered, our study is the first time to the best of our knowledge that a normalisation is proposed for prospect choices consistently with divisive normalisation reported in neurobiological studies. The divisive normalisation is considered as a canonical neural computation applied by the brain to make neural representations more effectively account for the encoded information about sensory input or higher-order mental constructs. Unsurprisingly, divisive normalisation has been reported to provide parsimonious explanation for complex phenomena in perceptual and cognitive studies, particularly when neural activity at different levels of brain organisation is dependent on the context, i.e., other sources of information or inputs, than the input of interest [22]. The notion of contextualisation is central to cognitive processes with decision-making as a primary representative [15, 17]. It provides a bridge between the efficient task demand dependent encoding of neural information through normalisation and observable behavioural parameters, e.g., value of outcomes. This has been demonstrated by Louie et al. [23] where context-dependent choice behaviour was correlated with the modulation of neural activity through divisive normalisation in monkeys' lateral interparietal cortex. In a different study by Li et al. [24] a gain-modulated (normalised) decision variable was shown to be accounted for by changes in blood-oxygen-level-dependent signals reflecting neural activity in a dorsal network of the human brain.

In other words, contextualisation of decision-making relates to the divisive normalisation of behaviourally relevant (latent) variables such as value, as reflected in the pattern of modulation of their neural correlates. This explains the flexibility of the valuation in decision-making across a wide range of situations and contexts, providing insights into the underlying neuro-computational mechanisms. It also casts light on the interpretation of the choice parameter β as the gain parameter during valuation that can dynamically control the interaction between different options. The gain can be related to neural activity through the canonical divisive transformation (see Louie et al. [15]): $\mu_i = K\frac{V_i}{\sigma_H + \sum_j \omega_j V_j}$, where V_i is the value of the prospect i under consideration (i.e., $i \in \{\mathrm{A}, \mathrm{B}\}$), μ_i is the mean firing rate representing the value of prospect i, K is a gain parameter, σ_H is the semi-saturation, and ω_i are weight terms (the summation is over all prospects). As shown by Louie et al. [15], the most important dependence of μ_i is on the sum of valuations, i.e., σ_H and ω_i have a comparably small effect. The L1 norm $V_n = |V_\mathrm{A}| + |V_\mathrm{B}|$ proposed in eq. (1) is thus a simplified form of divisive normalisation. The normalised utility has to be bounded just as its neural correlates (e.g., the range of neurophysiologically feasible/available firing rates). Thus, the cRUM proposes a *neurobiological* (rather than an *economic*) contextualisation of RUM, based on our current understanding of neurocomputational mechanisms with canonical computations as divisive normalisation.

Derivation of the choice parameter β. We validate cRUM and test the endogenous property of the choice parameter β by evaluating the model's generalisation capability in different choice scenarios from the literature. Specifically, we aim at capturing two scenarios: the first takes into account variability among decision-makers in a population, meaning that we expect the probability of P_{A} to vary between decision-makers; the second instead considers an average decision-maker in a population, meaning that we consider an aggregated (or population-representative) probability of choice P_{A}. Variability in a population can be captured by assuming a stochastic model for β with pdf defined on $(0, +\infty)$; in this work we assume that β follows a log-normal distribution, denoted here by $\beta \sim LN[\mu, \sigma]$, where μ and $\sigma > 0$ are the mean and standard deviation of the natural logarithm of β. We consider the average of the distribution, denoted by $E[\beta]$, to serve as a population-representative value for the choice parameter β, fixed across decision-makers.

In the process of estimating the β distribution, we impose constraints on its average and on its plausible range that accounts for a sufficiently high level of certainty. Specifically, following utility normalisation in eq. (1) and the proposed formalisation of intention, we suggest that β ought to reflect how choice certainty (the highest probability of choosing one prospect and the lowest probability of choosing the other prospect) is perceived by humans in terms of probabilities. We note that in eq. (1) $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} = -1$ or $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} = 1$ are limiting attitude differences (or preferences) for choosing B over A, or A over B, respectively. This means that $P_{\mathrm{A}} = 1/(1+\mathrm{e}^{\beta})$ or $P_{\mathrm{A}} = 1/(1+\mathrm{e}^{-\beta})$ correspond to sure choices of B or A, defined as choices associated with sure probabilities $P_{\mathrm{A}} = 0$ or $P_{\mathrm{A}} = 1$, respectively. Theoretically, sure choices $P_{\mathrm{A}} = 1$ or $P_{\mathrm{A}} = 0$ imply $\beta \to +\infty$; however, in prospect experiments, $P_{\mathrm{A}} = 1$ or $P_{\mathrm{A}} = 0$ correspond to sure probabilities as perceived by humans. Common notions of an *unlikely*, *improbable*, or *unexpected* event would be perceived differently and assigned different numerical values by decision-makers in a population; translation of such perceptions into numerical values and their variability between individuals have yet to be explored at depth [25, 26]. In this manuscript we consider human-social probability perception evidence, in particular perceptual numerosity data and data on social perception of risk/unlikely events, to infer plausible numerical values for perceived probabilities of unlikely events in a population of decision-makers.

Based on the human-social probability perception evidence available in the literature, we impose two constraints on the distribution of β. First, to hypothesise a population-representative value $E[\beta]$ for the choice parameter β, we observe that in *perceptual numerosity experiments* a qualitatively recognisable change in proportion is around $1/1\,000$ [26, 27, 28, 29, 30], which may be interpreted as the lowest proportion still perceived as non-zero by subjects. Thus, we infer $1/1\,000$ as the most plausible representative value of perceived probability of an unlikely event by a population of decision-makers, from which we deduce $E[\beta] \approx 7$ as the average value for the choice parameter (eq. (5), Methods). If the mean of the β distribution, constrained to 7, represents average perception of unlikely events in a population, a 95% confidence interval can be used to impose a lower bound on perceptions of unlikely events among individuals, and thus derive μ and σ in the pdf of β. We refer to *human perception of societal risks* from which we identify 1/1 000 000 as a generally accepted frequency threshold for (in)tolerable risk, both from the individual and societal risk perspectives (Methods). From these considerations, our second constraint is that most individuals in a population (say at least 95%) would perceive a zero probability threshold at roughly 1/1 000 000. From the two constraints, the distribution $\beta \sim LN[1.8, 0.5]$ can be deduced (eq. (6), Methods; Supplementary Fig. 2), modelling variability in a population of decision-makers.

The distribution $\beta \sim LN[1.8, 0.5]$ suggests a simple statistical model of the choice probability (eq. (8)). Typified curves for a population cumulative distribution function (cdf) of the choice probability P_{A} are plotted in Fig. 2. For $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} \to 0$, a step function at $P_{\mathrm{A}} = 1/2$ is obtained; while as $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} \to 1$ (resp. $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} \to -1$) the deterministic limit (or step function) indicating the sure choice of A (resp. B) is recovered.

Application: Experimental results on discrete choices datasets

First test of cRUM: Variability among decision-makers and the framing effect. We identify suitable studies for testing cRUM and variability within a population from the literature on the framing effect. We consider the pioneering neuroeconomics study by De Martino et al. [18] and the recent experiments conducted by Diederich et al. [31] (denoted as DS-FR1 and DS-FR2 in Table 1, respectively) to observe the variability of framing effect among decision-makers.

According to PT[1], the framing effect is defined as *contradictory attitudes toward risks involving gains and losses* [32] or as the tendency to prefer a sure over a risky option when a problem is framed in terms of potential gains instead of potential losses. Variability among decision-makers is captured by diverse exhibited behaviours, risk-avoidant vs risk-seeking choices, based on framing of the problem. Given that framing is a systematic behaviour captured by PT, the aim of the experiments reported in [18] was to reveal its neurobiological basis. In the experiments, 20 subjects chose between a risky (A) and sure (B) prospect: prospect pairs were designed with a shifted reference point in order to capture framing, and by construction the amounts received in the sure option were identical to the expected value of the risky option. The difference in probability of choosing the risky option in gain and loss frames (linearly related to a "rationality index") was determined for each subject and correlated with brain's neural activity. The neuroimaging results revealed a special

[1] In this manuscript we use the notation PT to refer also to Cumulative Prospect Theory [9].

modulatory role of amygdala, primarily associated with the emotional processes. The complex entanglement between endogenous effects is noted by De Martino et al. [18, p. 686–687] *[..] frame-related valence information is incorporated into the relative assessment of options to exert control over the apparent risk sensitivity of individual decisions.* In other words, biases in valuation related to risk and loss/gain, are combined internally with perception of "control". Thus the observed difference in "rationality" between the 20 subjects could be due to bias as well as variability in the perceived control. In the proposed cRUM, this implies that the observed framing effect variations among the 20 subjects could in principle be explained by variability between decision-makers in the choice parameter β, given utilities V_A and V_B for risky and sure prospects, respectively.

To predict variations among subjects observed in framing experiments [18, 31], we propose a simulation scenario using the log-normal distribution $\beta \sim LN[1.8, 0.5]$ derived above as a stochastic model of the variability of the choice parameter β (eq. (6), Methods). In particular, we drew 1000 samples of 20 simulated decision-makers from the underlying log-normal distribution of β and computed the probability P_A for gain/loss prospect pairs; the difference in P_A in the gain and loss frames quantifies the modelled framing effect (details in Supplementary Information). With the mean of $\zeta_\mathrm{A} - \zeta_\mathrm{B}$ being equal to -0.07 in the gain frame and to 0.07 in the loss frame (blue dashed curves in Fig. 2, left panel), the predicted median framing effect (i.e., difference in P_A at the median or 50th percentile between loss and gain frames) is around 22.2% (Fig. 2, right panel), with the corresponding observed median framing effect in [18] being 17.1%. This means that in a population sample a value around 20% is predicted to be the most likely outcome for the median framing effect, consistent with observations.

cRUM with the proposed β distribution reproduces remarkably well the variability in framing effect observed in [18] (Fig. 3(a), blue curve, Pearson correlation coefficient $r = 0.985$ when comparing the mean model output against data). Nevertheless, the difference between modelled and observed framing effect depends also on given utilities and thus on PT parameters $(\gamma, \delta, \lambda)$ in the computation of subjective valuations (eq. (4), Methods), especially on the exponent of the utility function δ and the exponent in the weighting function γ (Fig. 3(a), right panel). In short, an increase in δ implies less bias (explained as higher "rationality" in [18]) and hence the P_A difference between frames is smaller (Fig. 3(a), right panel, green curve). The risk perception parameter γ has a similar but weaker effect (Fig. 3(a), right panel, red curve). Finally, the effect of the loss aversion parameter λ is negligible (Fig. 3(a), right panel, yellow curve). Our focus is on the endogenous property of β as applied to the framing experiments in [18] where no direct calibration of PT parameters was provided. For simplicity we shall therefore use the "standard" PT parameters $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$ proposed in the original work on PT [9].

The proposed model for β captures also the effect of proportion of total amount offered in the sure prospect B, by definition equal to the probability of winning the gamble (Fig. 3(b), left panel): the observed values in both gain and loss frames are reasonably well reproduced by cRUM with $\beta \sim LN[1.8, 0.5]$ (distributed chart). The framing effect is stronger for higher probabilities of winning the gamble (60% and 80% in [18]), corresponding to larger amounts offered in the sure prospect, compared to smaller probabilities of winning the gamble (20% and 40% in [18]), corresponding to smaller amounts offered in the sure prospect. Comparatively, the effect of the initial amount of money offered to the 20 subjects observed in [18] is smaller (Fig. 3(b), right panel), while in cRUM the probability of risky choice P_{A} does not depend on the initial amount of money offered due to the normalisation (as proven in Remark 1 in the Supplementary Information; Supplementary Fig. 3).

The weak dependency of the probability of choosing the risky prospect A on the initial amount offered to participants has also been observed in the experimental studies on framing effect conducted by Diederich et al. [31]. These experiments consider a higher number of subjects as well as additional factors in the analysis, such as time constraints and induced need. cRUM with the proposed stochastic model for β captures reasonably well the observed variability in framing effect among decision-makers in the experimental settings proposed in [31], with correlation values higher than 0.95, even when comparing the obtained results with stochastic models for β estimated using standard calibration techniques from machine learning (Supplementary Fig. 4).

Second test of cRUM: Average representation of a population of decision-makers. In the next step of testing cRUM and the endogenous property of β, we consider 12 experiments reported in the literature (datasets DS1–DS12 in Table 1). Note that in general the analysis reported in the experimental studies did not include calibration and testing of RUM. A total of around 2 000 data points were considered, where each data point was obtained by aggregating choices from a number of decision-makers. Since DS1–DS3 have relatively few data points and constitute the classical references for PT, we group these into an aggregated data group, called DGs (with a total of 15 data points). For reference, data groups DGgl, consisting of DS1–DS10 and DS12 (1 095 data points, predominantly gain and loss prospects), and DGm, consisting of DS11 and DS12m (654 data points, purely mixed prospects), were also considered (Table 2(a)). For completeness we consider three scenarios of PT parameters $(\gamma, \delta, \lambda)$ used in the computation of valuations

V_{A} and V_{B} (Table 2(b)): PT parameters all equal to 1 corresponding to expected utility (EU), standard PT parameters, and calibrated PT parameters (values reported in Table 1).

To model aggregated choices we apply cRUM with a population-representative fixed value of β corresponding to the estimated mean of the distribution, $E[\beta]$. Although we argue for $E[\beta] = 7$, as derived here from experimental data on perceptual numerosity and social perception of extremely unlikely events (eq. (5)), for completeness we also test other mean values as population-representative, i.e., $E[\beta] \in \{1, 2, \ldots, 20\}$, while still preserving the constraint in the confidence interval. Statistical measures r (Pearson's correlation), MSE (mean squared error), and p-value of the t-test between data and cRUM model predictions are illustrated in Fig. 4(a) for PT parameters as in Table 1 (Supplementary Fig. 5(a) and Supplementary Fig. 6(a) for EU and standard PT parameters, respectively). To indicate high correlation and low MSE we consider correlation values r greater than 0.6 and MSE values lower than 0.05, respectively. The percentage of datapoints, aggregated across all datasets of Table 1, for which the correlation (resp. MSE) is greater (resp. lower) than given threshold values is illustrated in Fig. 5(a) for all PT parameters scenarios of Table 2(b). Although there is a notable difference in statistical measures between datasets and datagroups with predominantly gain/loss prospects (DS4–DS7, DS12, DGs), mixed prospects (DS11, DS12m, DGm), and gain/loss/mixed prospects (DS8–DS10, DGgl), a plausible range for the population-representative value of β is between 4 and 10 (Fig. 4(a)). This range corresponds to roughly 68% confidence interval of the distribution $\beta \sim LN[1.8, 0.5]$. For $E[\beta] \in [4, 10]$ the statistical indicators are $r > 0.6$ and MSE < 0.05, except for the two mixed prospect datasets DS11 and DS12m (Fig. 4(a)). In particular, cRUM with $E[\beta] = 7$, i.e., the mean of the hypothesised distribution inferred from the probability perception data [27] (eq. (5)), provides a reasonable population-representative value for predictions of gain and loss prospect choices (linear regression plots in Fig. 5(b), bottom row, and Fig. 6, to compare with the plots of Supplementary Fig. 7, bottom row, and Supplementary Fig. 8, respectively, obtained with $E[\beta] \in \{4, 10\}$).

Using standard PT parameters $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$ in the calculation of utilities $V_{\mathrm{A}}, V_{\mathrm{B}}$ instead of the specifically calibrated PT parameters for each dataset (i.e., the values of PT parameters reported in Table 1) causes a decrease in the value of correlation r for the datagroup DGgl (linear regression plots in Fig. 5(b), middle row; Supplementary Fig. 9). Moreover, neglecting the effect of bias by setting EU parameters $(\gamma, \delta, \lambda) = (1, 1, 1)$ (Table 2(b)), thereby assuming that decision-makers choose the outcome associated with maximum expected utility, reduces significantly correlation values r and increases MSE for most of the datasets/datagroups (Fig. 5(b), top row; Supplementary Fig. 10).

Ambiguity of prospects affects choices. Formulation of a decision problem (in this case prospects) is known to affect choices, e.g., complex vs simple or well-defined vs ill-defined problems [8]. Recent evidence shows that subjects prefer simpler prospect formulations [33]. In our present analysis, a clear distinction is made between mixed prospects and gain/loss prospects. Appraisal of alternative outcomes, resulting attitudes and intention to choose A or B (Fig. 1) depend on how well subjects comprehend prospect options. The notion of control of choice certainty perceived by a decision-maker, quantified by β, can be understood for a rational decision-maker as, among others, reflecting problem comprehension. A problem perceived with higher ambiguity should imply lower β, hence we anticipate lower β for mixed prospects compared to generally more comprehensible gain or loss prospects.

We propose three ambiguity indicators, given utilities $V_\mathrm{A}, V_\mathrm{B}$ and observed choice probability P_A: the median of the cdf of β, computed using eq. (9), the percentage of data points for which $\beta = 0$ (corresponding to $V_\mathrm{A} = V_\mathrm{B}$, or, equivalently, $P_\mathrm{A} = 1/2$), and the percentage of data points for which $\beta < 0$. We calculate and illustrate the ambiguity indicators for each dataset/group and for each PT parameters scenario in Table 2(b) (Fig. 4(b), Supplementary Fig. 5(b), Supplementary Fig. 6(b)). A negative β implies that $P_\mathrm{A} > 1/2$ for $V_\mathrm{A} < V_\mathrm{B}$, while a symmetric distribution of β around $\beta = 0$ is a statistical expression of the flip-of-coin case $P_\mathrm{A} = P_\mathrm{B} = 1/2$ as $\beta \to 0$. DS5 and DS9 have an unusually high number of cases for which $\beta = 0$, 12 out of 72 and 21 out of 108 data points, respectively; excluding DS5 and DS9, the overall fraction of data with $\beta = 0$ is 1.2%, whereas with DS5 and DS9 it is 4.6%, which is still comparatively small (Fig. 4(b), top right panel). The β medians for most data sets/groups are clustered in the 68% confidence interval of the proposed distribution for β (shadowed area in Fig. 4(b), left panel) for gain/loss prospects; clear exceptions are mixed prospects of DS11 and DS12m, where medians are closer to zero and a significant fraction of β values computed from eq. (9) are negative (Supplementary Fig. 4(b), bottom right panel). If biases are neglected, the fraction of negative β increases for a number of datasets/groups (Supplementary Fig. 5(b)), suggesting that biases as captured by PT are an integral part of comprehension and valuation.

Discussion

The novel cRUM with contextualised exogenous utilities and exogenous choice parameter β improves predictability of prospect choices across experimental settings significantly. Moreover, the separation between exogenous and endogenous variables facilitates interpretability of the novel choice parameter β as control parameter, capturing choice certainty or problem comprehension by decision-makers.

Our findings however also reveal limitations and potential sources of uncertainty. Choice predictability in behavioural economics [8] can be related to broader predictability challenges in psychology [34, 35] where probability of deciding on action A from several alternatives (A, B, C,...) can be written as $P(\mathrm{A}|X^*, X)$ with X^* and X being vectors of endogenous and exogenous variables for alternative options [8], respectively. The main difficulty when predicting decisions among alternative options in any context is to define and characterise X^*, or infer $f(X^*|X)$, the conditional pdf of endogenous variables or parameters [8]. Although the normalisation of utilities in cRUM removes explicit dependence of the rationality or free parameter on exogenous variables (prospect outcomes) in RUM, our results suggest implicit dependence of β on exogenous variables in terms of problem formulation or types of prospects: more ambiguous mixed prospects imply lower β compared to gain/loss prospects. In other words, dependence of $f(X^*|X)$ on X may be more complex than previously thought. Further studies are needed to identify endogenous mechanisms that control β and their dependence on problem formulation, possibly leading to new or modified existing psychological tests for estimating β in a population. Recent works by Erev et al. [36] and Peterson et al. [13], where different variants of choice problem formulation were considered, provide valuable experimental methodologies as well as a wealth of data for deeper characterisation of $f(X^*|X)$ in general and $f(\beta)$, indicating the pdf of β, in particular. Incorporating β in neuro-cognitive studies may also help to better understand the entanglement between valuation and perceived control or problem comprehension in prospect choices noted by [18].

Although traditionally economic choice models are static, as in eq. (1), experiments provide evidence of changes in choices in repeated trials [36, 13], and theoretical dynamic decision-making models that explain the effect of repeated prospect choices are available in the literature [20, 37, 38]. cRUM has potential to be extended to account for sequential effects in choice behaviour. It can also be incorporated into an interconnected network of decision-makers to study collective action and social influence, the latter being an important part of TPB (Fig. 1) neglected in this study.

Methods

Contextualisation of the RUM choice function

With utilities (or, valuations) V_A and V_B for prospects A and B, respectively, one way of incorporating contextualisation in valuation of prospects into a Fechner type choice function [10] is proposed in this work as

$$P_{\mathrm{A}} = F\big(\beta(\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}})\big) = F\left(\beta\,\frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{V_n}\right) \tag{1}$$

and similarly for P_{B}, where $\beta(\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}})$ is referred to as *intention* of choosing A over B, the contextualised utility $\zeta_{\mathrm{A}} = V_{\mathrm{A}}/V_n$ is referred to as *attitude* toward option A, $V_n = |V_{\mathrm{A}}|+|V_{\mathrm{B}}|$ is the L1 norm for the vector $[V_{\mathrm{A}}, V_{\mathrm{B}}] \in \mathbb{R}^2$, β is a positive scalar parameter, and $F(\cdot)$ is an S-shaped continuous function. The novelty of the choice probability estimate formulated in cRUM (eq. (1)) lies in the L1 normalisation, as a method to draw comparisons between the intentions towards the two alternative prospects, A and B. In this new context, we refer to β as the choice parameter; β can be interpreted as quantifying control, i.e., how sure an individual is when making a choice: $\beta \to 0$ implies $P_{\mathrm{A}} = \frac{1}{2}$, and $\beta \to +\infty$ implies $P_{\mathrm{A}} = 1$ for $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} > 0$ and $P_{\mathrm{A}} = 0$ for $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} < 0$.

The most common form of $F(\cdot)$ in the RUM is Logit [6, 10, 7, 13], i.e., $F(y) = (1 + e^{-y})^{-1}$, which yields

$$P_{\mathrm{A}} = \left(1 + \exp\left(-\beta\,\frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{V_n}\right)\right)^{-1} = \big(1 + \exp\big(-\beta(\zeta_A - \zeta_B)\big)\big)^{-1} = 1 - P_{\mathrm{B}}, \tag{2}$$

where V_{A} and V_{B} primarily depend on exogenous variables (nominal values, probabilities of outcomes).

Valuation of options. Prospects A and B are defined in terms of outcomes and respective probabilities, and can be written as:

$$\mathrm{A} : \{(Y_{1,\mathrm{A}}\,, \pi_{\mathrm{A}}), (Y_{2,\mathrm{A}}\,, 1 - \pi_{\mathrm{A}})\} \qquad \mathrm{B} : \{(Y_{1,\mathrm{B}}\,, \pi_{\mathrm{B}}), (Y_{2,\mathrm{B}}\,, 1 - \pi_{\mathrm{B}})\} \tag{3}$$

where $Y_{1,\mathrm{A}}$ and $Y_{2,\mathrm{A}}$ are outcomes for prospect A, with probabilities π_{A} and $1-\pi_{\mathrm{A}}$, respectively, and similarly for prospect B. For each prospect, we have $Y_1 > Y_2 \geq Y_0$ for gain (or positive) prospects, $Y_1 < Y_2 \leq Y_0$ for loss (or negative) prospects, while $Y_1 < Y_0 < Y_2$ for mixed prospects; Y_0 is a reference value typically set to 0. A subject will choose a prospect based on the perceived values of alternatives A and B, denoted respectively by V_{A} and V_{B} in this work: eq. (2) implies that option A is preferred to B if $V_{\mathrm{A}} \geq V_{\mathrm{B}}$. According to PT [9], V_{A} and V_{B} are computed as

$$V(Y_1, Y_2) = \begin{cases} U(Y_1)\,w(\pi) + U(Y_2)\,(1 - w(\pi)), & \text{for positive or negative prospects} \\ U(Y_1)\,w(\pi) + U(Y_2)\,w(1 - \pi), & \text{for mixed prospects} \end{cases} \tag{4a}$$

$$U(Y) = \begin{cases} (Y - Y_0)^{\delta+}, & \text{if } Y \geq Y_0 \\ -\lambda(Y_0 - Y)^{\delta-}, & \text{if } Y < Y_0 \end{cases} \qquad \text{where } Y = Y_1, Y_2 \tag{4b}$$

$$w(\pi) = \frac{\pi^\gamma}{\big(\pi^\gamma + (1 - \pi)^\gamma\big)^{1/\gamma}}, \tag{4c}$$

where V, Y_1, Y_2, and π pertain to A or B. In eq. (4), $U(Y)$ is the utility function and $w(\pi)$ is the decision weighting function. The PT parameters $(\gamma, \delta, \lambda)$ in eqs. (4b)–(4c) can be inferred from experiments (see Data description and Table 1 for details on the datasets considered in this study).

The choice parameter β

The intention of choosing option A over B is bounded in the interval $[-\beta, \beta]$: in view of eq. (1), this means obtaining $P_{\rm A} = 1$ for $\beta(\zeta_{\rm A} - \zeta_{\rm B}) = \beta$, and $P_{\rm A} = 0$ for $\beta(\zeta_{\rm A} - \zeta_{\rm B}) = -\beta$. In other words, for rational subjects, $\zeta_{\rm A} - \zeta_{\rm B} = -1$ should imply *zero* probability of choosing A. The concept of zero probability needs to be somehow related to *human-social perceptions of certainty*, i.e., it must depend on what humans *perceive* as near-zero probability or, equivalently, probability of an unlikely event. Assuming that the perception of near-zero probabilities by humans varies across a population of decision-makers, we ask the following questions: (i) How does the perception of unlikely events vary between decision-makers? (ii) What is a representative value of lowest perceived probability of an unlikely event in a population of decision-makers? Since the near-zero probability is captured in cRUM by the lower limit $\beta(\zeta_{\rm A} - \zeta_{\rm B}) = -\beta$, to answer these questions means to hypothesise a distribution for β to model variability in perceived probability of an unlikely event between decision-makers, whose mean represents an average perceived probability of unlikely events by a population of decision-makers.

In the following, we deduce a distribution for β from what is generally accepted in the society as a representative probability of an unlikely event where we seek a threshold, or cutoff as a prevailing, generally acceptable probability perception for an unlikely event (or near zero-probability event). We consider a log-normal distribution a stochastic model β in a population with pdf defined on $(0, +\infty)$, denoted here by $LN[\mu, \sigma]$ where μ and σ are the mean and standard deviation of $\log(\beta)$. To obtain estimates of μ and σ, we shall first establish a population-representative value, mean of the distribution denoted by $E[\beta]$, and a lower limit of perceived probability for unlikely events, within say a 95-percentile, which in view of eq. (2) sets an upper bound on β.

Derivation of a population-representative value

Consider risk criteria that are used for management of hazardous activities or actions in societies [39, 40, 41, 42]. The main distinction for establishing such criteria is made between individual and societal risk of casualties or fatalities per year. Whereas societal risk considers rare events with potentially large number of fatalities (e.g., accidents), individual risk is focused on a single individual exposed constantly to a hazard. In the literature, activities with a fatality risk less than 1/1 000 are acceptable or tolerable, whereas over 1/1 000 are unacceptable or intolerable. When computing the societal risks using the so-called Farmer's diagram, the anchor frequency for 1 fatality is most often set to 1/1 000 [39]. However, tolerable individual risk per year is typically defined as 1/1 000 000 [39, 40].

An important societal perception of frequencies or probabilities is related to genetic disorders referred to as rare diseases. To stimulate drug development with tax incentives and government funding, US Congress passed the Orphan Drug Act in 1983 for rare disease defined as a condition affecting less than 200 000 US citizens [43] for a total US population at the time of around 230M; hence approximately 1/1 000 is considered by US society as a threshold for rare vs non-rare diseases. Japan [44] and the EU [45] have taken similar initiatives for stimulating drug development, where the threshold frequency for rare disease was defined somewhat lower than the US, as 1/2500 and 1/2000, respectively.

Finally, visual perceptions of probability as frequencies have been studied experimentally [30, 27], where the resolution considered is around 1/1 000, i.e., 1/1 000 is the lowest perceived probability. Subjects are tasked to estimate the proportion of coloured discs in boxes with a total of 1 000 discs; this implies a probability resolution of 1/1 000. An identical experimental setup to [27] with the same probability resolution of 1/1 000 was considered recently in [28], where the authors tested trial-by-trial updating models of probability perception. Furthermore, the probability resolution of around 1/1 000 was also considered as a lower bound in studying risk and probability perception in children [46]. Zhang and Maloney [29] studied probability and frequency distortion in perception and action, presenting a wide range of data both from the literature and own experiments, which are also based on subjects' estimates of probabilities from a box with 600 discs of two colours.

From the above considerations, we conclude that approximately 1/1 000 is a reasonable representative frequency threshold distinguishing human-social perceptions of unlikely vs likely events, hypothesising 1/1 000 as a representative or average cut-off for zero probability. Thus, in view of eq. (2), a population-representative value is computed as follows, where log indicates the natural logarithm:

$$E[\beta] = \log\left(\frac{1 - 1/1\,000}{1/1\,000}\right) = \log(999) \approx 7. \tag{5}$$

Derivation of a stochastic model

In the previous section we consider $E[\beta] = 7$ as a representative value for a population. In this section we include variability across individuals, by proposing a stochastic model for β. In the proposed log-normal β distribution we constrain the mean to be equal to 7 and we impose a constraint on the 95% confidence interval. If the mean represents average perception of unlikely events in a population, a 95% confidence interval is used to impose a lower bound on perceptions of unlikely events across individuals.

Considering the Farmer's diagrams for societal risk, we find that a number of countries define a cutoff frequency as 1/1 000 000, a good example being Hong Kong [47], the UK [48], and China [49], such that risks less than 1/1 000 000 are considered tolerable. Thus 1/1 000 000 seems to be a generally accepted frequency threshold for (in)tolerable risk, both from the individual and societal risk perspectives. Note that 1/1 000 000 coincides with current best estimate of natural hazard mortality rates; any hazard with frequency below 1/1 000 000 does not require any action. Based on above considerations, it appears reasonable that most individuals in a population (say at least 95%) would perceive a zero probability threshold at roughly 1/1 000 000 whereby, together with imposed mean equal to 7, $\mu = 1.8$ and $\sigma = 0.5$, can be deduced (details in Supplementary Information).

We then postulate the stochastic model for the choice parameter β as:

$$\beta \sim LN[1.8, 0.5]. \tag{6}$$

Separation of exogenous and endogenous variables. Eq. (1) is aligned with the extended random utility framework of [8]. The probability of choosing, say, option A is conditioned on exogenous (X) and endogenous (X^*) variables as $P_{\mathrm{A}} \equiv P(\mathrm{A}|X, X^*)$; endogenous variables are also referred to as internal or latent (unobservable). Given a separable parametrisation for the endogenous variables X^*, the observable unconditional probability of choosing A [8] is extended to our case as

$$P(\mathrm{A}|X) = \int_{X^*} P(\mathrm{A}|X, X^*) f(X^*|X) dX^* = \int_0^\infty P(\mathrm{A}|X, \beta) f(\beta|X) d\beta \tag{7}$$

where the last term can be approximated by the population-representative value (eq. (5)), and $f(X^*|X)$ is a pdf for the endogenous variables.

The framing effect and variability in a population of decision-makers. To evaluate variability between decision-makers in a population and test the proposed stochastic model for β we consider two experimental studies on the framing effect, the pioneer work of De Martino et al. [18] (DS-FR1 in Table 1), detailed here, and the extensive study of Diederich et al. [31] (DS-FR2 in Table 1), detailed in the Supplementary Information which contains also a comparison between the proposed stochastic model (eq. (6)) and calibrated models for β.

The work of De Martino et al. presents experimental data that combine framing and variability effects, obtained by studying variation between 20 subjects. In the experiments the subjects were tasked with choosing between risky (A) and sure (B) prospects, presented in the context of gain or loss frames. Prospect pairs were designed with a shifted reference point in order to capture framing. Specifically, the prospects were defined from the following (16 possible) combinations of outcomes Y and probabilities π:

$$Y_i = (£25, £50, £75, £100), \quad \pi_j = (1/5,\, 2/5,\, 3/5,\, 4/5), \quad i, j = 1, 2, 3, 4$$

$$\begin{cases} \mathrm{A_G} : \{(Y_i, \pi_j), (0, 1-\pi_j)\}, & \mathrm{B_G} : \{(Y_i \pi_j, 1), (0, 0)\} \quad \text{(gain frame)} \\ \mathrm{A_L} : \{(-Y_i, 1-\pi_j), (0, \pi_j)\}, & \mathrm{B_L} : \{(-Y_i(1-\pi_j), 1), (0, 0)\} \quad \text{(loss frame)} \end{cases}$$

with L and G denoting loss and gain frames, respectively. The framing effect, i.e., the difference between loss and gain frames in probability of risky choices, was found in the study to be in a range between 6% and 40% for each one of the 20 subjects (Fig. 3(a)): that is, each subject chose more often the risky option (A) when prospects were framed as losses than when prospects were framed as gains, although the actual monetary outcome was identical. The subject with the lowest value ($\approx$ 6%) was interpreted as the most rational individual and the subject with highest value ($\approx$ 40%) as the least rational individual.

pdf of choice probability P_A. Using the proposed stochastic model for β and given $\zeta_\mathrm{A} - \zeta_\mathrm{B}$, the pdf of choice probability P_A, $f(P_\mathrm{A}|\zeta_\mathrm{A} - \zeta_\mathrm{B})$, can be obtained from eq. (2). Given $P_\mathrm{A} = F(\beta(\zeta_\mathrm{A} - \zeta_\mathrm{B}))$ and $f_\beta(\beta)$ as the pdf of β, $f(P_\mathrm{A}|\zeta_\mathrm{A} - \zeta_\mathrm{B})$ can be calculated as $\left|\frac{\partial}{\partial P_\mathrm{A}} F^{-1}(P_\mathrm{A}, \zeta_\mathrm{A} - \zeta_\mathrm{B})\right| f_\beta(F^{-1}(P_\mathrm{A}, \zeta_\mathrm{A} - \zeta_\mathrm{B}))$, yielding:

$$f(P_\mathrm{A}|\zeta_\mathrm{A} - \zeta_\mathrm{B}) = \frac{1}{(1-P_\mathrm{A})P_\mathrm{A}|\zeta_\mathrm{A} - \zeta_\mathrm{B}|} f_\beta\left(\log\left(\frac{P_\mathrm{A}}{1-P_\mathrm{A}}\right)\frac{1}{\zeta_\mathrm{A} - \zeta_\mathrm{B}}\right). \tag{8}$$

Complex choice scenarios and ambiguity indicators. The cRUM with the stochastic model for β we propose, while performing reasonably well given its simplicity, is unable to explain more complex choice scenarios, including, e.g., choice prospects with multiple outcomes, mixed prospects, and in general characterised by poor problem comprehension by decision-makers. In what follows we infer a pdf for β from observed P_A and $\zeta_\mathrm{A} - \zeta_\mathrm{B}$ and we propose ambiguity indicators identifying complex choice scenarios.

An expression for inferring β from observed P_A and $\zeta_\mathrm{A} - \zeta_\mathrm{B}$ is obtained from eq. (2):

$$\beta = \frac{1}{\zeta_\mathrm{A} - \zeta_\mathrm{B}} \log\left(\frac{P_\mathrm{A}}{1-P_\mathrm{A}}\right). \tag{9}$$

In particular, $\beta > 0$ if $(P_\mathrm{A} - \frac{1}{2})(\zeta_\mathrm{A} - \zeta_\mathrm{B}) > 0$. However, using observed P_A and $\zeta_\mathrm{A} - \zeta_\mathrm{B}$ data from the datasets of Table 1 and the datagroups of Table 2(a) to infer β using eq. (9), we obtain zero or even negative values for β (Fig. 4(b)). In view of the deterministic limit $P_\mathrm{A} = \frac{1}{2}$ for $\beta = 0$, we consider $\beta = 0$ and $\beta < 0$ as suitable ambiguity indicators of a choice problem; a greater fraction (in absolute value) of $\beta < 0$ in an experimental sample would indicate greater overall difficulty for subjects to value outcomes. In addition to the fraction of zero and negative β values, another proposed ambiguity indicator is the β medians in the cdf curves for inferred β (eq. (9)).

Data description

Table 1 summarises the 12 datasets (denoted by DS1–DS12) and 2 framing-effect datasets (denoted by DS-FR1–DS-FR2), collected from the literature, which provide the evidence base for our study. The number of data points for each dataset varies between 2 and 559, with a total number of data points close to 2 000. Each data point defines two prospects, A and B, in terms of outcomes and respective probabilities (eq. (3)), and associated observed choice probability P_{A} for A and $P_{\mathrm{B}} = 1 - P_{\mathrm{A}}$ for B; note that the choice probability for a given prospect was obtained aggregating choices made by many subjects. Of particular interest are the more recent works of Erev et al. [50, 21, 36] (DS6, DS8, DS9 in Table 1) with extensive data where a number of variants in the prospect formulation were considered. For our purpose however, only the simplest prospect cases are extracted such that compatibility is ensured with earlier studies (DS1–DS8, Table 1). The experiments of Lopes and Oden [51] (DS5) have multiple outcomes which are either gains or losses. DS11 has multiple outcomes and mixed prospects as the most complex formulation of options. Prospects used in the experiments of Erev et al. [36] (DS9) and Peterson et al. [13] (DS12) were generated using the algorithm proposed in [36], but for this study we extract only the baseline cases. The specific steps of data extraction are detailed in the Supplementary Information.

As a reference and to test cRUM across datasets, we consider three aggregated data groups (Table 2(a)). The data group DGs aggregates the first three datasets of Table 1 and has a total of 15 data points; it represents classical datasets of prospect theory, where the standard PT parameters are introduced (see also the next section and Table 2(b)). The data group referred to as DGgl contains roughly 1 100 data points, with predominantly gain (g) and loss (l) prospects; it aggregates the datasets DS1–DS10 and DS12, i.e., all data except DS11 and DS12m, which consist of pure mixed prospects. Note that DS8, DS9, and DS10 contain gain, loss, and mixed prospects in roughly equal proportions. Finally, the data group DGm gathers the datasets DS11 and DS12m (with a total of 654 datapoints), i.e., it consist of mixed prospects.

PT parameters used in this study. Selection of $(\gamma, \delta, \lambda)$ in eq. (4) quantifies different risk and valuation biases. In this work we analyse effect of variations of PT parameters by considering three scenarios of increasing complexity, as illustrated in Table 2(b).

The first scenario corresponds to *expected utility* (EU): $(\gamma, \delta, \lambda) = (1, 1, 1)$, which implies a simple expected value without any biases (or, with negligible biases).

The second scenario identifies *standard PT parameters*: following the experimental data analysis of Tversky and Kahneman in [9], standard PT parameter values are $\delta = \delta^+ = \delta^- = 0.88$ (for both gains and losses) and $\lambda = 2.25$ in eq. (4b). The exponent in the weighting function in eq. (4c) was reported as 0.61 for positive and 0.69 for negative prospects. Subsequent studies (e.g., [51, 10]) obtained higher values of weighting function exponent. As a compromise and for simplicity, since this work does not aim at investigating calibration of PT parameters (utilities are considered exogenous variables) and we wish to limit the overall number of PT parameters setting the main focus on the choice parameter β, we use $\gamma = 0.65$ (average value) as the standard PT value for all prospects.

Finally, the third scenario reports *calibrated values of PT parameters*: along with the respective references, the PT parameters $(\gamma, \delta, \lambda)$ used in this work for predictive modelling are the ones reported in Table 1. If unavailable in the reference, standard PT parameter values, i.e., $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$, were used.

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Acknowledgements

Fundings This work was supported in part by the project Humanizing the Sustainable Smart City (hiss-digitalfutures.se) within Digital Futures.

Author contributions VC conceived and supervised the project; VC and AF collected the data; AF and VC analysed the data; VC, AF, PH, MM, KHJ discussed the results and wrote the manuscript.

Competing interests The authors declare no competing interests.

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Data and material availability All results are reported in the main text and Supplementary Information. If accepted, all the data used in this study will be made available through a repository site or through supplementary data files. Should full access to data be required for peer review, the corresponding author will provide it.

Tables & Figures Tables 1 to 2; Figures 1 to 6.

Supplementary Information

Supplementary Text:

1. Problem formulation.
2. Contextualisation by means of normalisation: From RUM to cRUM.
3. Application: Experimental results on discrete choices datasets.

Supplementary Figs. 1 to 10.

Supplementary Table 1.

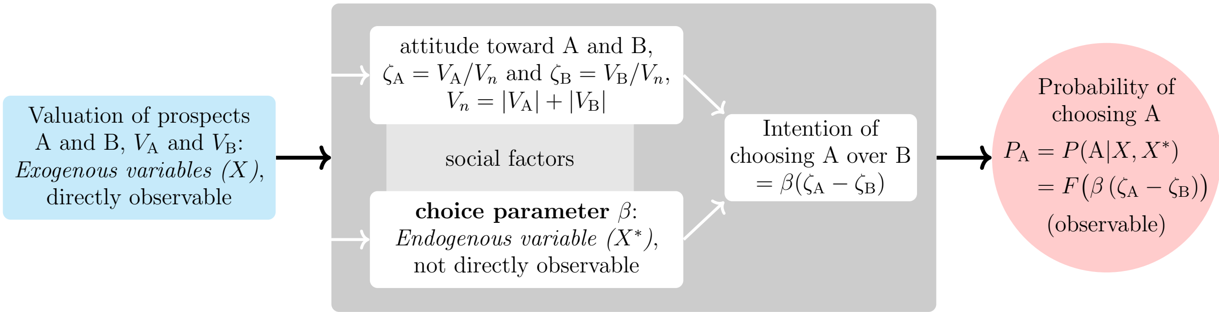

Figure 1: Illustration of cRUM, the proposed theoretical framework for predictive modelling of choice between prospects A and B, conceptualised using notions from TPB. Contextualisation of the valuation is ensured by the normalisation using L1 norm $V_n = |V_A| + |V_B|$. The separation between exogenous and endogenous variables follows the concept presented by Ben-Akiva et al. [8]. The key novelty in the framework is the choice function for computing the choice probability P_A. Exogenous variables (X) represent observable input information as outcomes, and probabilities for prospects A and B. Endogenous variables (X^*) account for bounded rationality due to limitation/effect of comprehension, limited cognitive flexibility, emotion, motivation, moral value, etc; they are not observable.

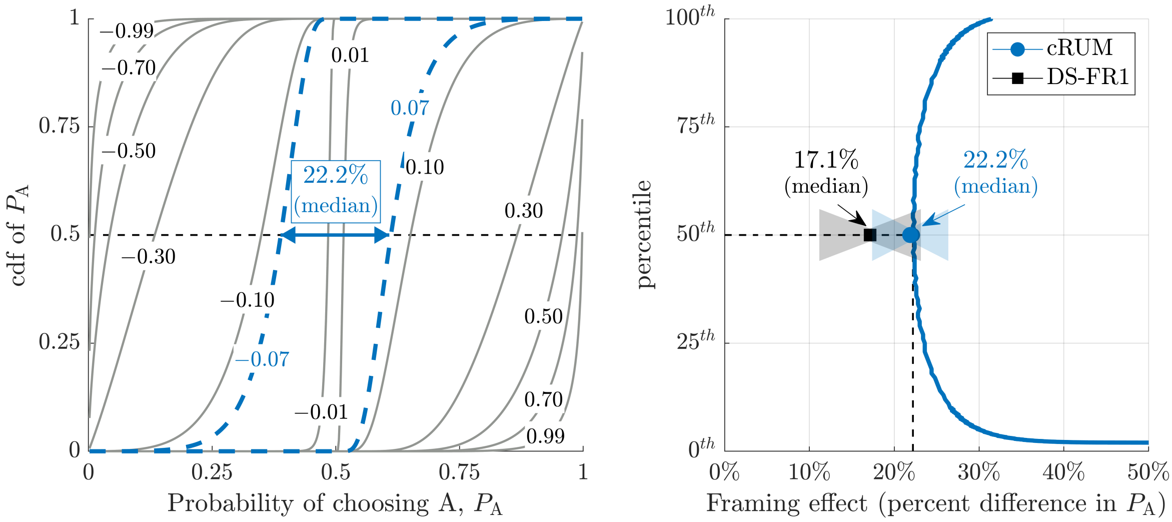

Figure 2: (left panel): cdf of P_A type-curves for fixed attitude toward choosing prospect A, deduced from eq. (8), given $\zeta_A - \zeta_B \in [-1, 1]$. Blue dashed curves are mean of experimental values $\zeta_A - \zeta_B$ from De Martino et al. [18] in both gain and loss frames; the dashed black thin line is the median (50th percentile). (right panel): The framing effect estimated with cRUM using the prospect pairs from De Martino et al. (DS-FR1 in Table 1), i.e., the difference between the blue dashed curves in the left panel. A blue circle and blue shadowed area indicate the median framing effect and its 95% confidence interval, respectively, obtained using cRUM with $\beta \sim LN[1.8, 0.5]$. A black square and black shadowed area indicate the median framing effect observed in [18] and its 95% confidence interval, respectively. Given the overlapping confidence intervals for the medians, we conclude that there is no statistical difference.

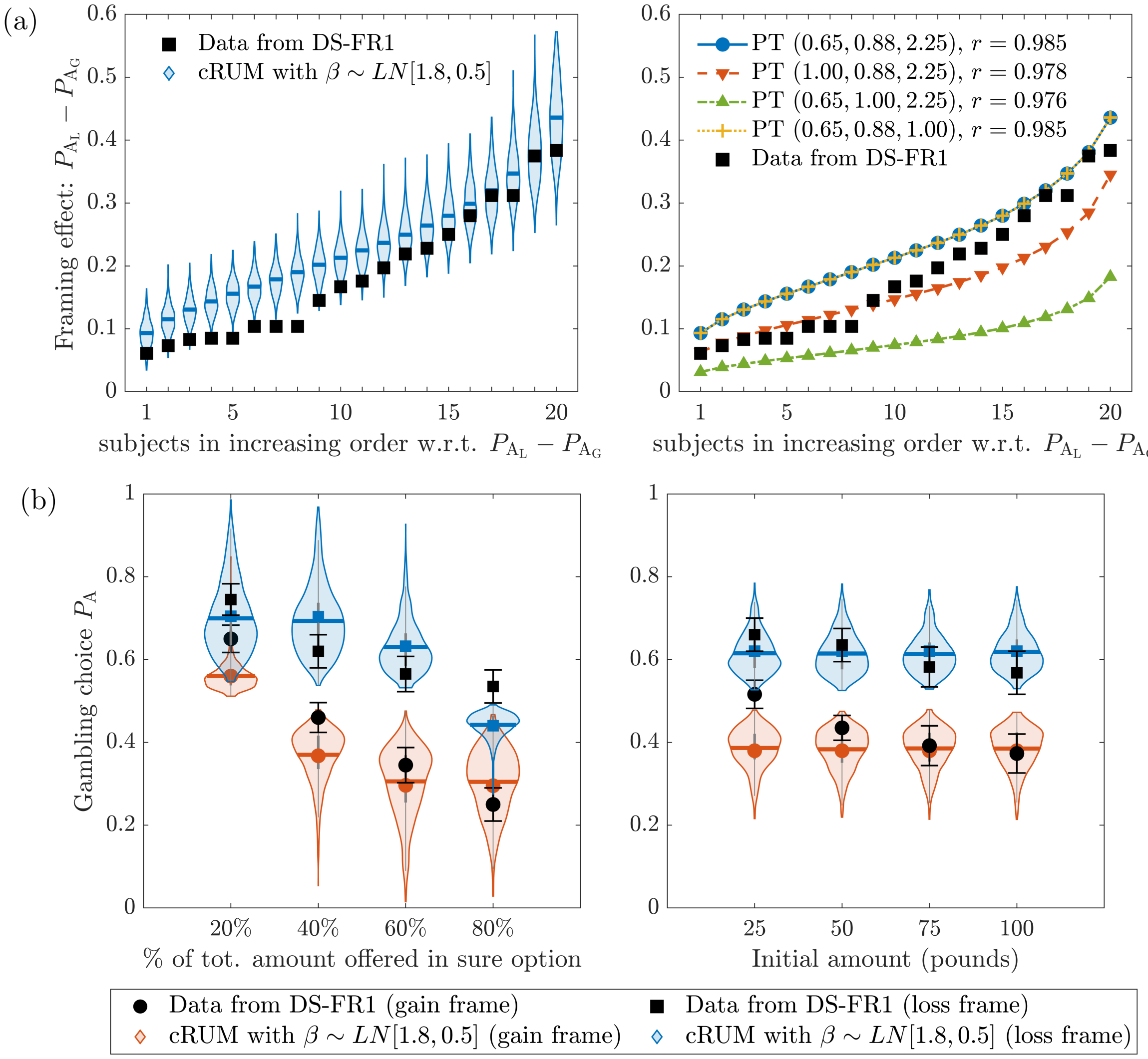

Figure 3: Framing and variability analysis using cRUM (eq. (2)) with the proposed stochastic model for β (eq. (6)); the data are from De Martino et al. [18] (DS-FR1 in Table 1). (a): 20 decision-makers in ascending order according to the framing effect, i.e., the percentage increase in their gambling choice in the loss frame relative the gain frame. The left panel illustrates distribution charts obtained by considering the proposed stochastic model $\beta \sim LN[1.8, 0.5]$ in eq. (2) for a total of 1000 samples of 20 simulated subjects. Calculations consider standard PT parameters, i.e., $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$. The right panel illustrates model sensitivity to PT parameters $(\gamma, \delta, \lambda)$, where the data is compared to modelled mean value; the symbol r indicates the associated values of Pearson's correlation between observed (black symbol) and modelled (coloured symbols) mean value of percentage increase in gambling choice. (b): Gambling choice probability P_A in loss and gain frames for different fractions of total amount offered in the sure option (left panel), and for different initial total amounts (right panel). Black symbols are data from De Martino et al. [18], distribution charts are obtained from cRUM with $\beta \sim LN[1.8, 0.5]$ with coloured symbols used to indicate the modelled mean value. Red colour indicates gain frame, while blue colour indicates loss frame. PT parameters are again $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$.

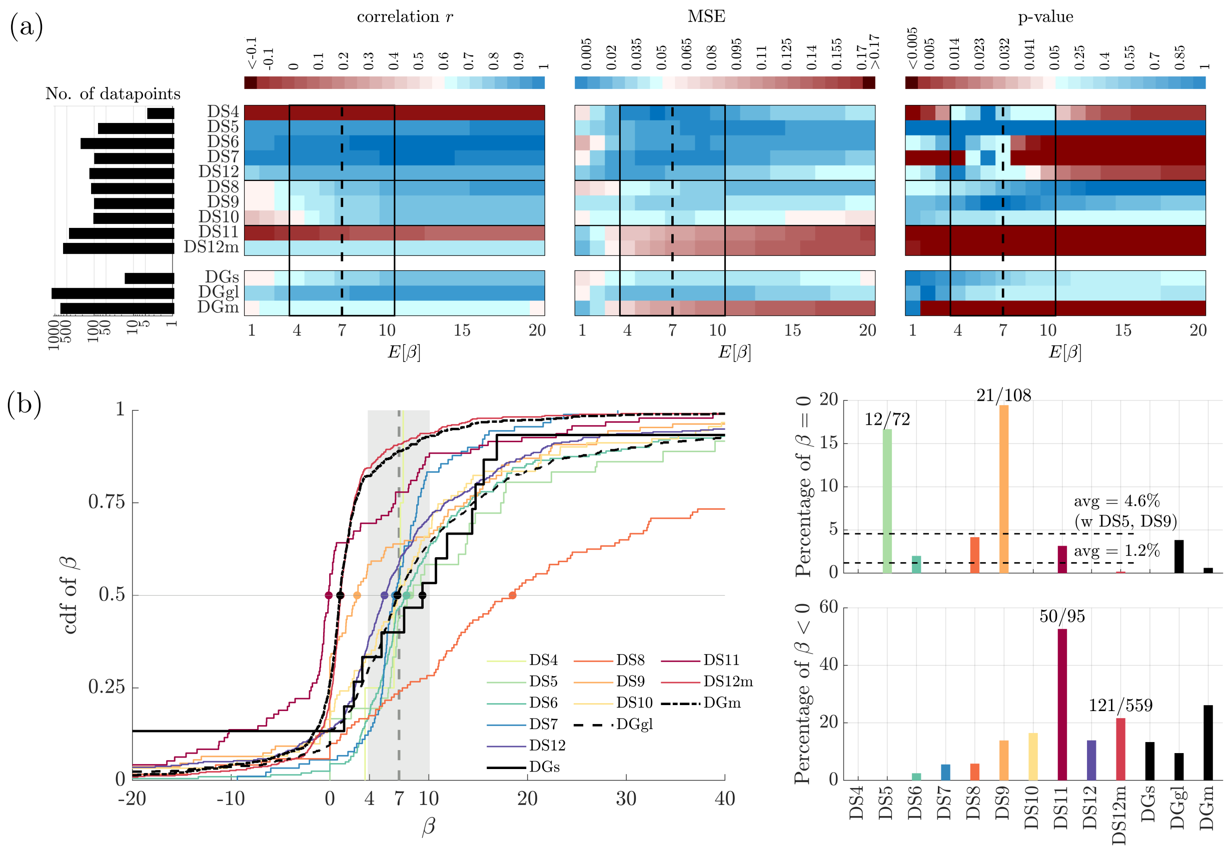

Figure 4: (a): Model fit/Comparison in terms of statistical measures (correlation r, mean squared error MSE, and p-value of t-test; Supplementary Information) between observed choice probability P_{A} and predicted choice probability P_{A} using cRUM with $E[\beta] \in \{1, 2, \ldots, 20\}$ and all datasets and datagroups listed in Tables 1 and 2(a). PT parameters are as in Table 1. Blue colour signals what we consider desirable, i.e., high correlation, low MSE, and high p-value. Overall (except DS11 and DS12m) the statistical indicators are $r > 0.6$, MSE< 0.05, and p-value $\not< 0.05$ (blue colour) when $E[\beta]$ belongs to the 68% confidence interval of $LN[1.8, 0.5]$, range illustrated by vertical black lines. (b): cdf of β obtained from eq. (9) (left), percentage of datapoints for which $\beta = 0$ (top right), and percentage of datapoints for which $\beta < 0$ (bottom right) for all datasets (colour-coded) and datagroups (black) of Tables 1 and 2(a). The proposed ambiguity indicators (Methods) are the percentage of zero and negative β values (right panels), and the β medians (indicated by dot symbols in the curves, left panel). DS5 and DS9 have the highest number of cases for which $\beta = 0$. DS11 and DS12m have the highest number of cases for which $\beta < 0$; moreover, the inferred β medians are negative (DS11) or close to zero (DS12m).

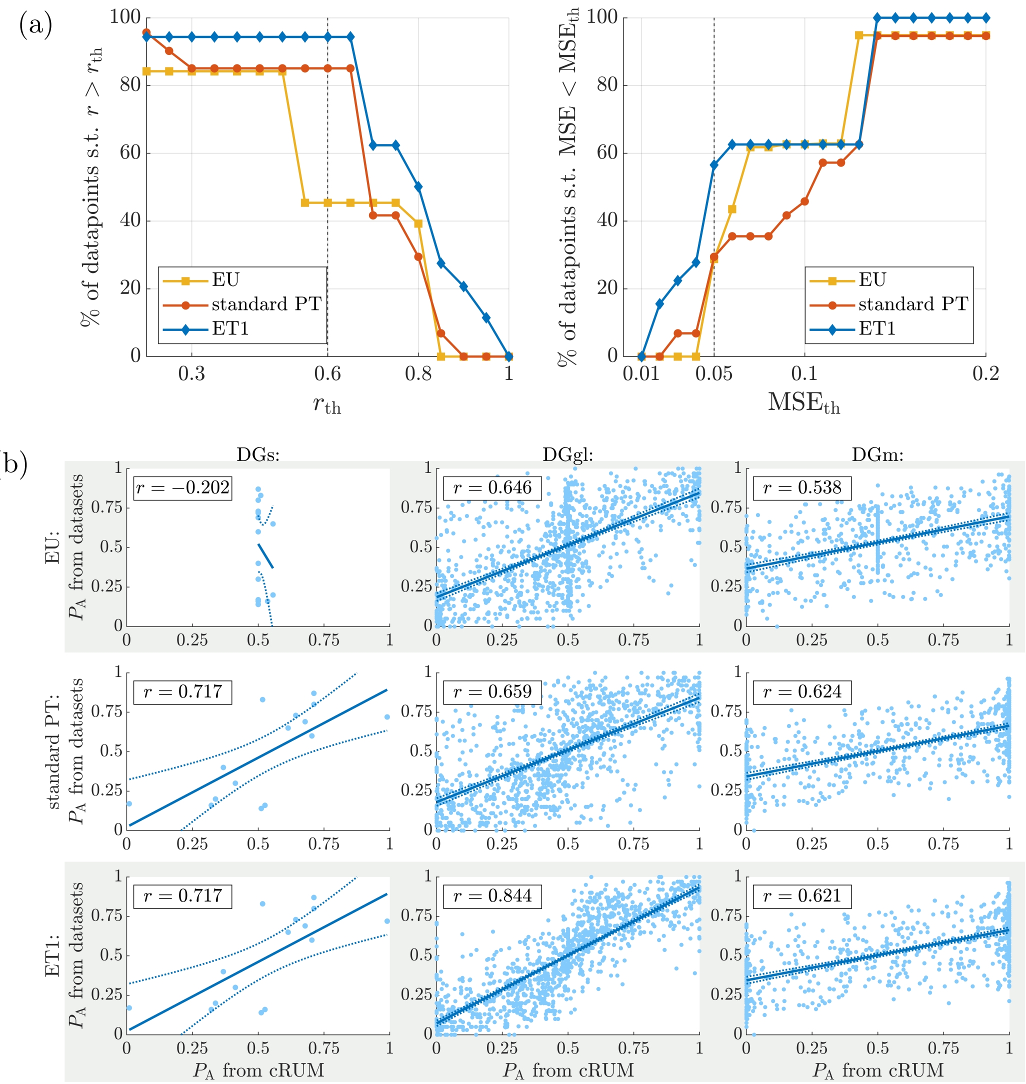

Figure 5: (a): Percentage of datapoints *across* datasets for which the correlation r is greater than a threshold r_{th} (left panel), and for which the MSE is less than a threshold MSE_{th} (right panel), for all PT parameters scenarios of Table 2(b). The dashed lines indicate the threshold values used to indicate high correlation and low MSE in Fig. 4(a). (b): Observed choice probability P_{A} vs estimated choice probability P_{A} using cRUM with $E[\beta] = 7$ and corresponding linear regression line for all PT parameters scenarios of Table 2(b) and all datagroups of Table 2(a) (described in the title of individual plots).

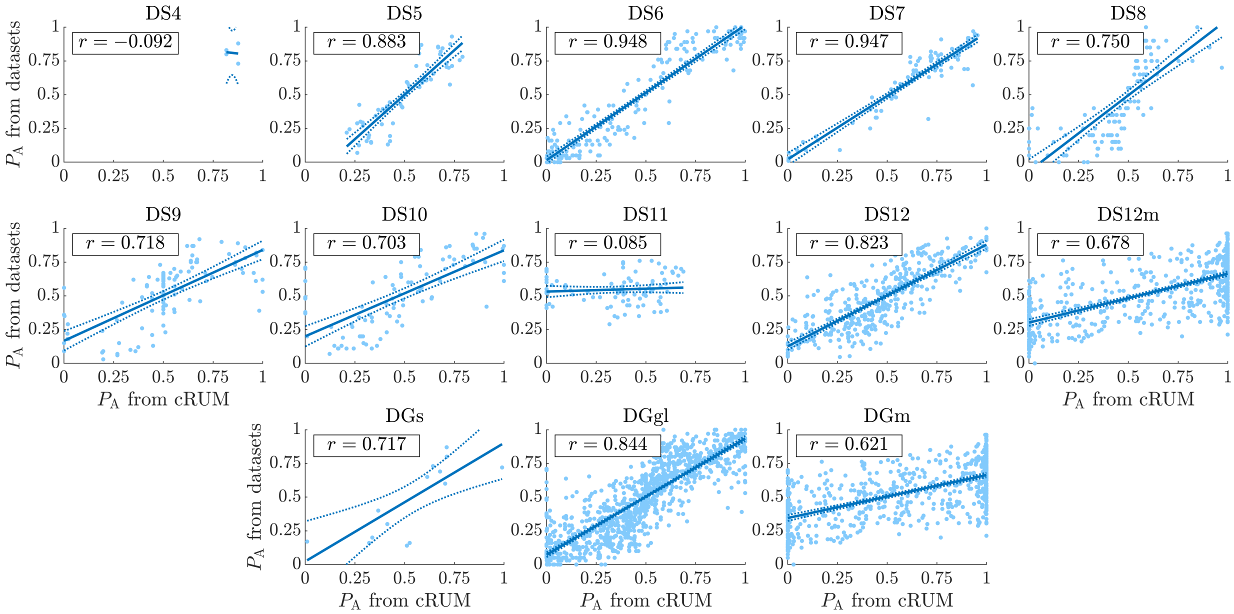

Figure 6: Linear regression plots between the observed choice probability P_{A} and the estimated choice probability P_{A} using cRUM with $E[\beta] = 7$ and corresponding correlation values r for each dataset of Table 1 and each datagroup of Table 2(a). The PT parameters used in the computation of utilities are from Table 1.

Data set (DS#)	**Experiment**	**Reference**	**No. of problems**	**PT parameters** $(\gamma, \delta, \lambda)$
DS1	gain/loss	Kahneman and Tversky (1979) [52]	11	$(0.65, 0.88, 2.25)^{\star,a}$
DS2	gain/loss	Tversky and Kahneman (1981) [32]	2	$(0.65, 0.88, 2.25)^{\star,a}$
DS3[b]	gain/loss	De Martino et al. (2006) [18]	2	$(0.65, 0.88, 2.25)^{c}$
DS4	gain/loss	Brandstätter et al. (2006) [53]	4	$(0.75, 0.33, 2.25)^{d}$
DS5	gain/loss (multiple outcomes)	Lopes and Oden (1999) [51]	72	$(0.75, 0.33, 2.25)^{d}$
DS6	gain	Erev et al. (2002) [50]	200	$(0.75, 0.33, -)^{\star,d}$
DS7	gain	Stott (2006) [10]	90	$(0.96, 0.19, -)^{\star}$
DS8	gain/loss/mixed	Erev et al. (2010) [21]	120	$(0.7, 0.89/0.98, 1.5)^{\star}$
DS9	gain/loss/mixed	Erev et al. (2017) [36]	108	$(1, 1, 1)^{e}$
DS10	gain/loss/mixed	Murphy and Brincke (2018) [54]	91	$(0.65, 0.88, 2.25)^{\star,f}$
DS11	gain/loss/mixed (multiple outcomes)	Brooks et al. (2018) [55]	95	$(0.65, 0.89/0.92, 1.69)$
DS12	gain/loss	Peterson et al. (2021) [13]	395	$(0.65, 0.88, 2.25)^{\star}$
DS12m	mixed	Peterson et al. (2021) [13]	559	$(0.65, 0.88, 2.25)^{\star}$

Data set (DS-FR#)	**Experiment**	**Reference**	**No. of problems for each frame**	**No. of subjects**
DS-FR1[b]	framing[c]	De Martino et al. (2006) [18]	16 (gain) + 16 (loss)	20
DS-FR2	framing[c]	Experiment 2, Diederich et al. (2020) [31]	48 (gain) + 48 (loss)	54

Table 1: Data sets collected from literature that constitute the evidence basis for validating eq. (1). (top): Across subjects studies (bottom): Within-subject studies. Legend. $\star$: PT parameters are explicitly calibrated in the corresponding reference. **a:** Following the experimental data analysis of Tversky and Kahneman in [9], our standard PT parameters are $\lambda = 2.25$ and $\delta = \delta^+ = \delta^- = 0.88$ for both gains and losses in eq. (4b). The exponent γ in the weighting function (eq. (4c)) was estimated to 0.61 and 0.69 for gain and loss prospects, respectively; for simplicity, we use $\gamma = 0.65$ (i.e., the average) as the standard PT value. **b:** DS3 = DS-FR1 is used to address variability and framing. **c:** Standard PT values are used since no other values are calibrated and given in the reference. **d:** In Lopes et al. (DS4) PT parameters from three different datasets (DS2, DS5, DS6 in our notation) are compared; accordingly to their results, we use PT parameters from DS6. **e:** In DS9 we test EU (i.e., PT parameters all equal to 1) as the simplest baseline and find that it works reasonably well, better than PT standard values. **f:** In Murphy and Brincke (DS10), PT parameters are calibrated as distributions. We take a simple approach and compare EU and PT standard values, finding that the latter work better.

(a)

Data groups (DG#)	**Datasets included (from Table 1)**	**Prospects' Types (predominant)**	**No. of data points**
DGs	DS1-DS3	gain/loss (standard PT param.)	15
DGgl	DS1-DS10, DS12	gain/loss	1095
DGm	DS11, DS12m	mixed	654

(b)

Scenario	**PT parameters** $(\gamma, \delta, \lambda)$	**Notation**
EU	$(1, 1, 1)$	expected utility
Standard PT	$(0.65, 0.88, 2.25)$	standard PT (from Tversky and Kahneman [9])
Calibrated	from Table 1	calibrated PT

Table 2: (a): (aggregated) Data groups considered in this work. (b): Scenarios considered in this work to test effect of variability of PT parameters on cRUM.